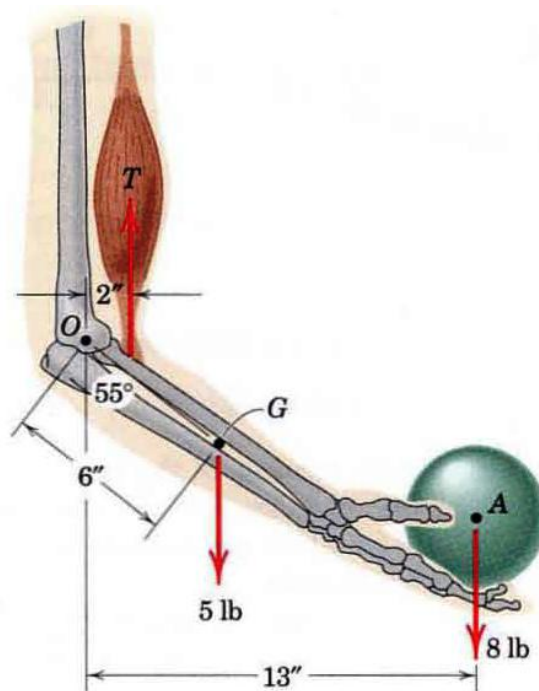# Mechanical Engineering Statics
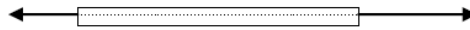
Assistant Prof. Dr. Arshed Abdulhamed
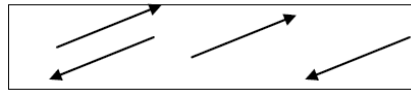
1.1   <u>Force Systems</u>

A force can be defined as the action of one body on another that changes/tends to changes the state of the body acted on.

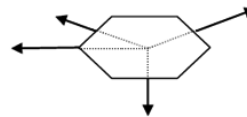Categories depending on their arrangement in plane:

- **Coplanar collinear forces**
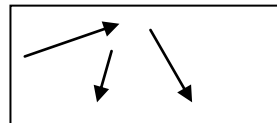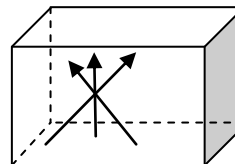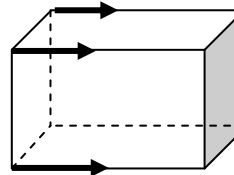
- **Coplanar parallel forces**

- **Coplanar concurrent forces**

- Non-concurrent Coplanar **forces**

- Concurrent Non- Coplanar

- Parallel Non- Coplanar

- Non Concurrent Non-parallel Non-coplanar

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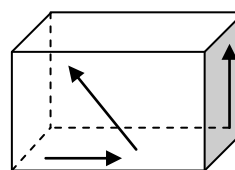
1.2   Conversion From F.P.S to S.I
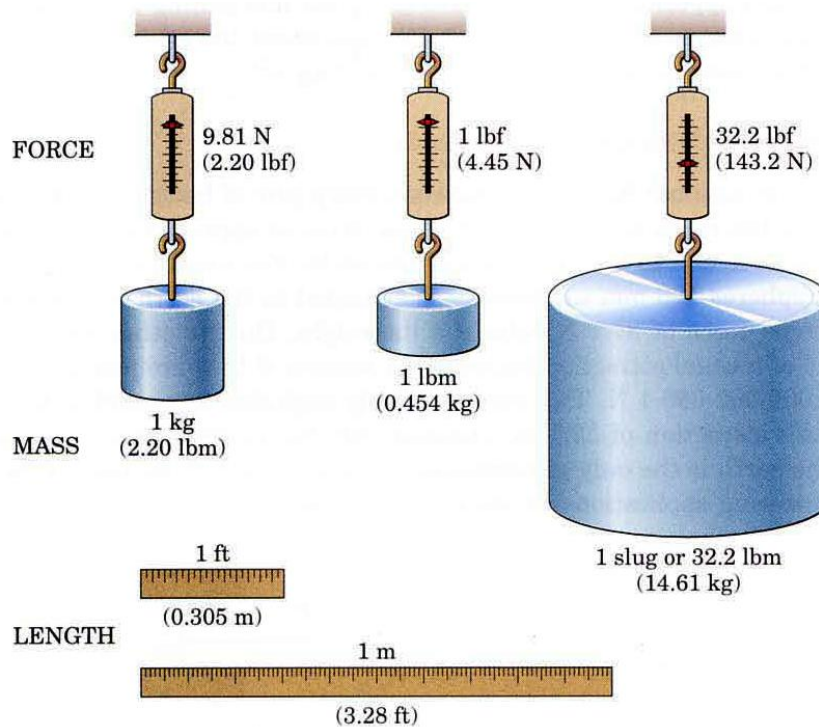
$1\ Mi = 5280\ ft\ \times 0.3045\ m = 1609\ m$

$1\ in = \dfrac{1}{12} ft = \dfrac{1}{12} \times 0.3045 = 0.0254\ m = 25.4\ mm$

$Mega\ N =\ 10^5\ N$

$Gega\ N =\ 10^9\ N$

$\mu =\ 10^{-6}\ N$

$1\ H.P = 0.746\ Kw$

1.3 Scalar and Vector

- **Scalars** are quantities that are fully described by a magnitude (or numerical value) alone.
  (distance, volume, temperature…)
- **Vectors** are quantities that are fully described by both a magnitude and a direction.
  (force, velocity, acceleration…)

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1.3 Parallelogram Law

If Q and P are two forces concurrent in point "o" the result of this forces is R

$$R = \sqrt{P^2 + Q^2 - 2QP\cos\theta}$$

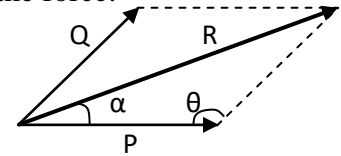The angle of the resultant is made between the resultant and onne of the force.

This angle can be determine by the low of sin

$$\frac{\sin \alpha}{Q} = \frac{\sin \theta}{R}$$

$$\alpha = \sin^{-1}\left(\frac{Q \sin \theta}{R}\right)$$

Example
Determine the resultant R of the two forces Q= 80N and P = 70N acting on the body and calculate the angle of the resultant.
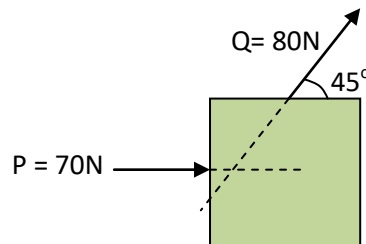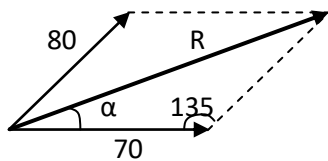
$$R = \sqrt{80^2 + 70^2 - 2 \times 80 \times 70 \times \cos135} = 138.6N$$

$$\frac{\sin \alpha}{70} = \frac{\sin 135}{R} \quad \rightarrow \quad \alpha = \sin^{-1}(\frac{70 \sin 135}{138.6}) = 20.918$$

H.W
Determine the resultant R of the two forces Q= 80N and P = 70N acting on the body and calculate the angle of the resultant

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1.3 Force and Components

It can be analysis the force in to two components

F_x …….. in x axis $F_x = F \cos \theta = F \sin \beta$

F_y……..... in y- axis $F_y = F \sin \theta = F \cos \beta$

$$F_x = F \cos \theta \qquad F = \sqrt{F_x{}^2 + F_y{}^2}$$
$$F_y = F \sin \theta \qquad \theta = \tan^{-1} \frac{F_y}{F_x}$$

Example: Find the x and y components for force F
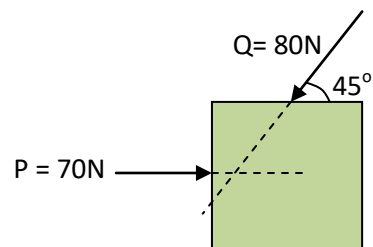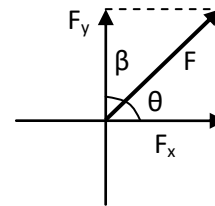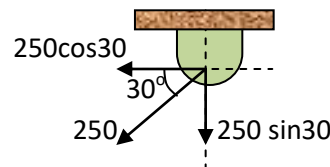
$F_x = 250 \cos 30 = 250 \sin 60 = 216.5 N$

$F_y = 250 \sin 30 = 250 \cos 60 = 125 N$

250cos30

30°

250   250 sin30

The forces $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.

Solution. The scalar components of $\mathbf{F_1}$, from Fig. a, are

$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad Ans.$$
$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad Ans.$$

The scalar components of $\mathbf{F_2}$, from Fig. b, are

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N} \qquad Ans.$$
$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N} \qquad Ans.$$

Note that the angle which orients $\mathbf{F_2}$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $\mathbf{F_2}$ is negative by inspection.

The scalar components of $\mathbf{F_3}$ can be obtained by first computing the angle α of Fig. c.

$$\alpha = \tan^{-1} \left[\frac{0.2}{0.4} \right] = 26.6°$$

Then $F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N} \qquad Ans.$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N} \qquad Ans.$$

Alternatively, the scalar components of $\mathbf{F_3}$ can be obtained by writing $\mathbf{F_3}$ as a magnitude times a unit vector $\mathbf{n}_{AB}$ in the direction of the line segment AB. Thus,

Helpful Hints

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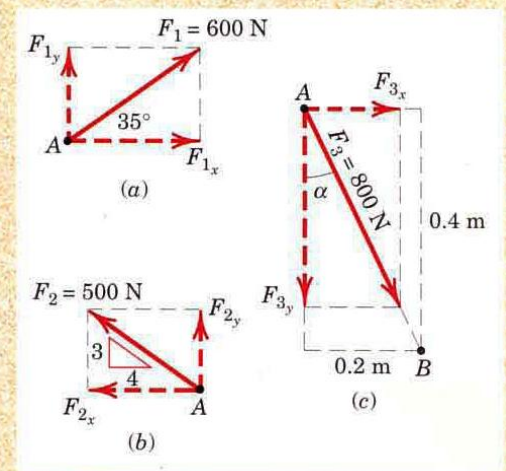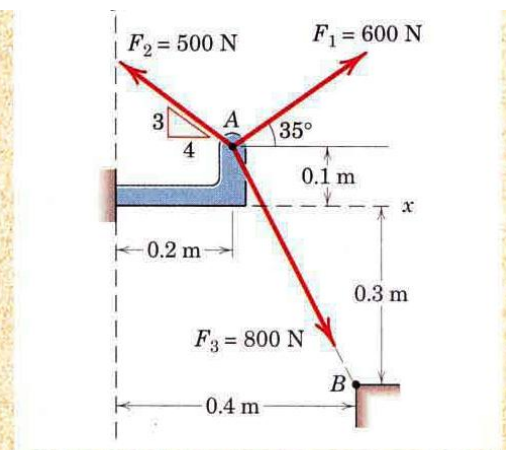
The y-component of the force **F** which a person exerts on the handle of the box wrench is known to be 70 lb. Determine the x-component and the magnitude of **F**.

Ans. F_x = 29.2 lb, F = 75.8 lb

H.W
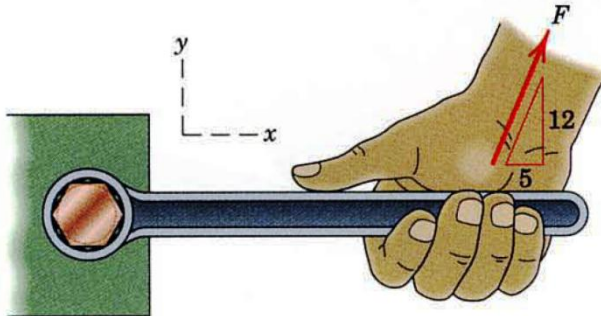
Problem 2/7

Determine the resultant **R** of the two forces shown by (a) applying the parallelogram rule for vector addition and (b) summing scalar components.
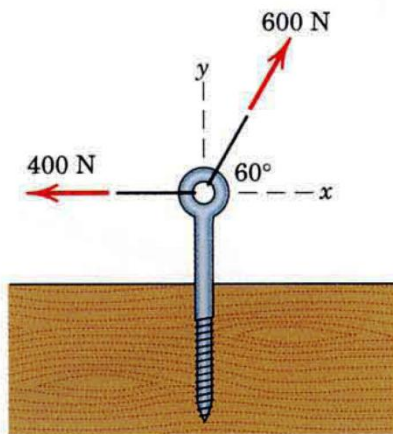
Problem 2/8

To satisfy design limitations it is necessary to determine the effect of the 2-kN tension in the cable on the shear, tension, and bending of the fixed I-beam. For this purpose replace this force by its equivalent of two forces at A, F_t parallel and F_n perpendicular to the beam. Determine F_t and F_n.

Ans. F_t = 1.286 kN, F_n = 1.532 kN

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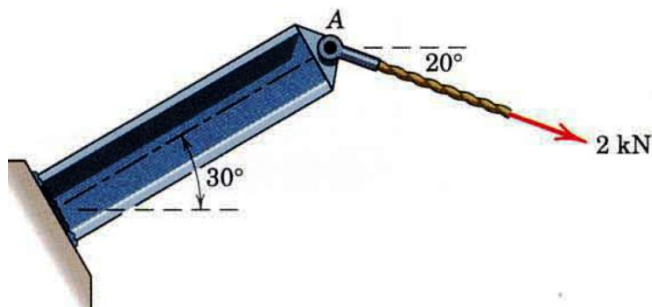
1.5 Resultant of Concurrent Force Components

If there are many concurrent components act on a body

R_x ……is the summation of forces on x-axis $R_x = \sum F_x$
R_y ……is the summation of forces on y-axis $R_y = \sum F_y$

R……..is the resultant $R = \sqrt{R_x^2 + R_y^2}$

$$\theta = \tan^{-1}(\frac{R_y}{R_x})$$

$$R_x = \sum \overrightarrow{F_x} = F_2 - F_1 \cos\theta$$
$$R_y = \sum \uparrow F_y = -F_3 - F_1 \sin\theta$$
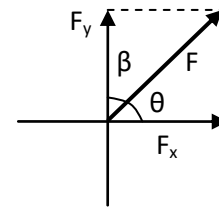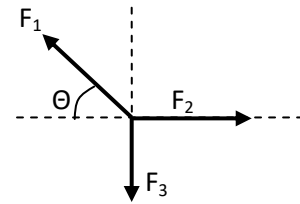
Example
Find the resultant of the two forces and its slope, as shown in the figure.

$$P_x = 200\cos\theta = 200 \times \frac{4}{5} = 160N$$
$$P_y = 200\sin\theta = 200 \times 4/5 = 120N$$

$$F_x = 300\cos\emptyset = 300 \times \frac{3}{5} = 180N$$
$$F_y = 300\sin\emptyset = 300 \times \frac{4}{5} = 240N$$
$$R_x = \sum \overrightarrow{F_x} = 160 + 180 = 340$$
$$R_y = \sum \uparrow F_y = 120 - 240 = -120$$
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{340^2 + 120^2} = 360.5$$
$$\tan\alpha = \frac{R_y}{R_x} \to \alpha = \tan^{-1}\frac{360}{340} = 19.44^0$$

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Example

An aircraft retaining hook is subjected to two forces as shown. Determine the magnitude of the resultant?

$$R_x = \sum \vec{F_x} = 150sin\theta - 200sin\beta$$

$$= 150 \times \frac{4}{5} - 130 \times \frac{12}{13} = 120 - 120 = 0$$

$$R_y = \sum \uparrow F_y = 150cos\theta - 130cos\beta$$
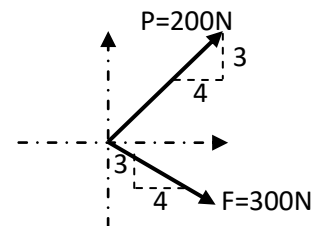
$$= 150 \times \frac{3}{5} - 130 \times \frac{5}{13} = 140N$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{140^2 + 0^2} = 140N$$
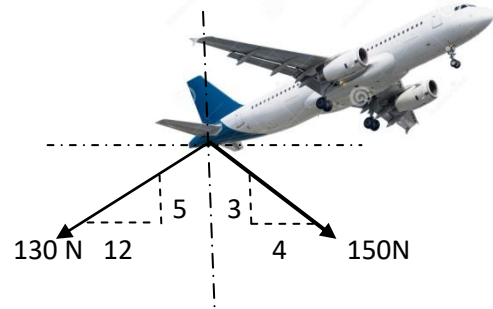
Example

The force F which acts on the block of figure has a horizontal rectangular component of 100Lb. Determine the rectangular component of F perpendicular to the inclined plane?
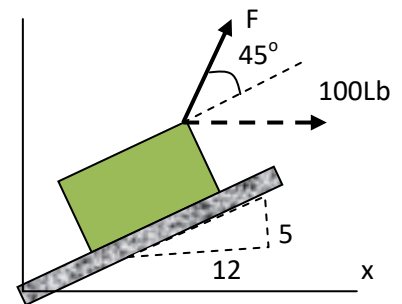
Example

At what angle θ must the 800-lb force be applied in order that the resultant **R** of the two forces has a magnitude of 2000 lb? For this condition, determine the angle β between **R** and the vertical.

8
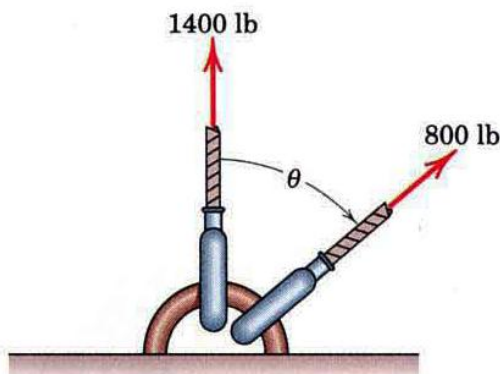
H.W

Find the magnitude of the force P. If you know the magnitude of the resultant is 100N and the magnitude of the force Q = 50N as shown

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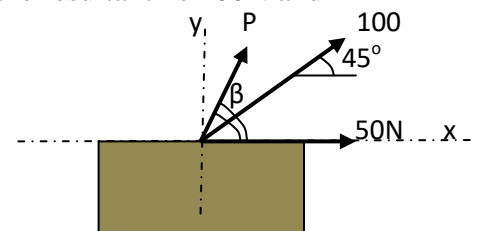
1.6 Moment of Force

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*. Figure 2/8*b* shows a two-dimensional body acted on by a force **F** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm d*, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as
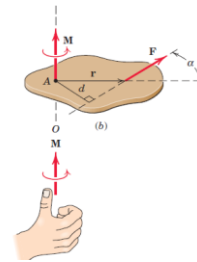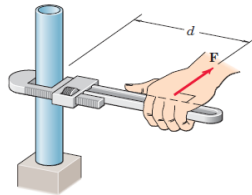
$M = F \times d$

JAMI LE I NUDLEM 2/J

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.

Solution. **(I)** The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(II) Replace the force by its rectangular components at A,

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

$$M_O = 460(4) + 386(2) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes

$$d_1 = 4 + 2 \tan 40° = 5.68 \text{ m}$$

and the moment is

$$M_O = 460(5.68) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

(IV) Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$$d_2 = 2 + 4 \cot 40° = 6.77 \text{ m}$$

and the moment is

$$M_O = 386(6.77) = 2610 \text{ N·m} \qquad \textit{Ans.}$$

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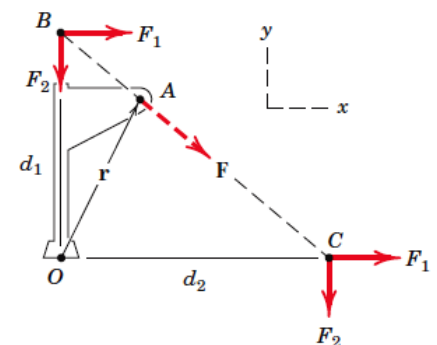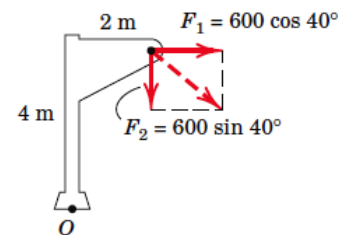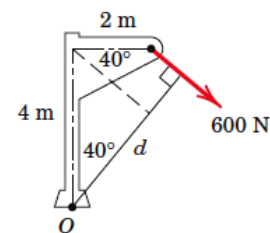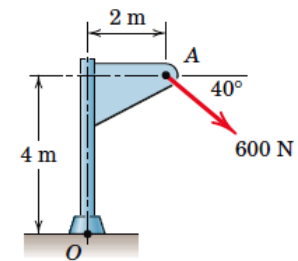
Example

Calculate the moment of the 250N force on the handle of the monkey about the center the bolt

$F_x = 250 \sin 15$, $\qquad F_y = 250 \cos 15$

$M_o = -250 \sin 15 \times 0.03 + 250 \cos 15 \times 0.2$

$M_o = 48.1\ N.m$
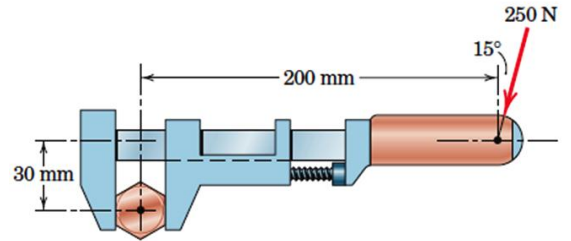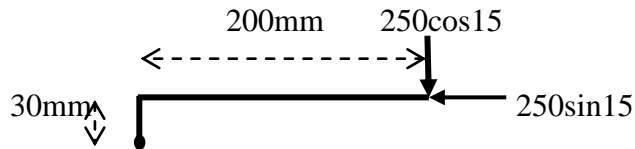
H.W

Calculate the moment of the 400-N force about point O in five different ways.

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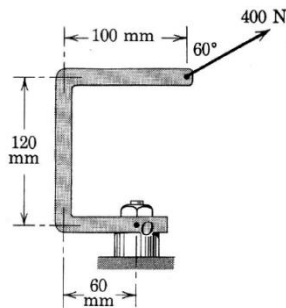
HW

The 30-N force **P** is applied perpendicular to the portion BC of the bent bar. Determine the moment of **P** about point B and about point A.
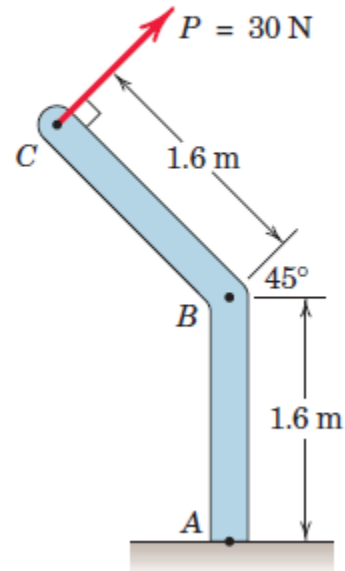
Elements of the lower arm are shown in the figure. The weight of the forearm is 5 lb with mass center at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the sphere. What must the biceps tension force be so that the overall moment about O is zero?

12
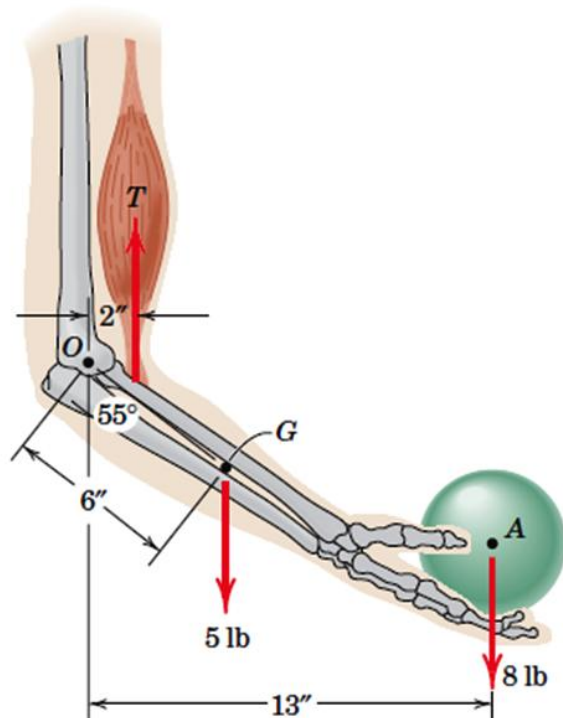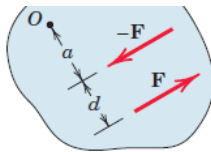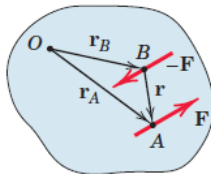
1.7 couple

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ a distance d apart, as shown in Fig. 2/10a. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple $\mathbf{M}$. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

$$M = Fd$$

Example

Replace the horizontal 80· lb force acting on the lever by an equivalent system consisting of n force at 0 and a couple.

$$M = 80 \times 9sin60 = 624 \, lb - in$$

Thus, the original force is equivalent to the 80· lb force at 0 and the 624-lb-in. couple as shown in the third of t he three equivalent figures .

80 lb

9"

60°

O

80 lb   80 lb

≡   ≡

O

80 lb   80 lb   80 lb

624 lb-in.

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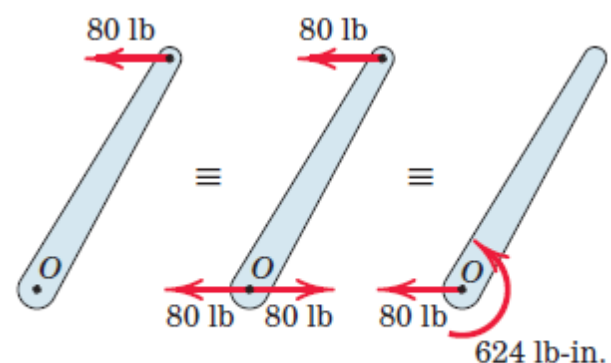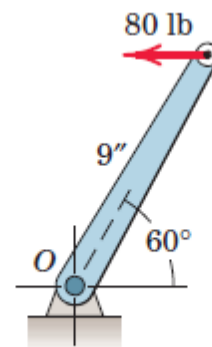
Compute the combined moment of the two 90-lb forces about (a) point O and (b) point A.
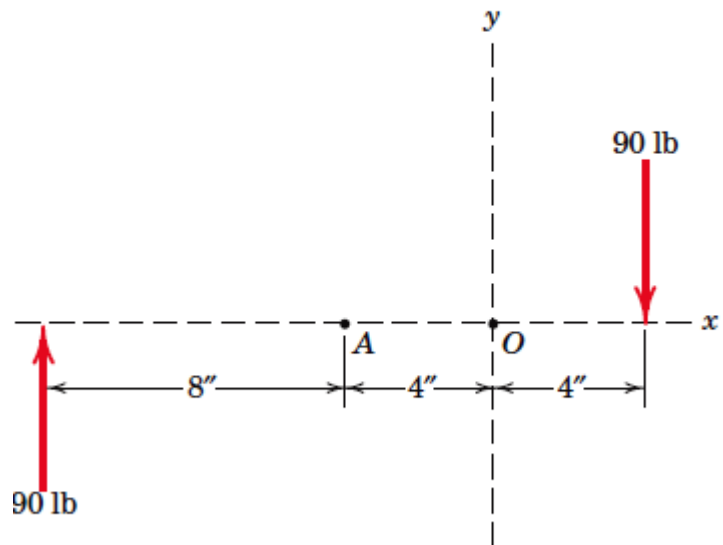
Problem 2/59

Replace the 12-kN force acting at point A by a force–couple system at (a) point O and (b) point B.

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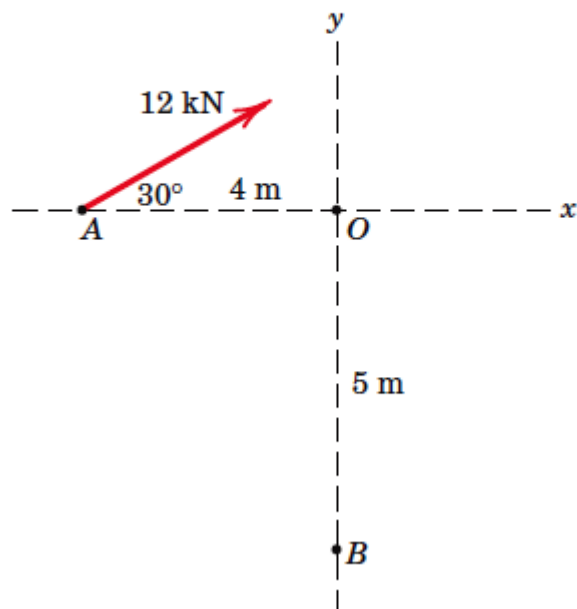
H.W

Each propeller of the twin-screw ship develops a full speed thrust of 300 kN. In maneuvering the ship, one propeller is turning full speed ahead and the other full speed in reverse. What thrust P must each tug exert on the ship to counteract the turning effect of the ship's propellers?
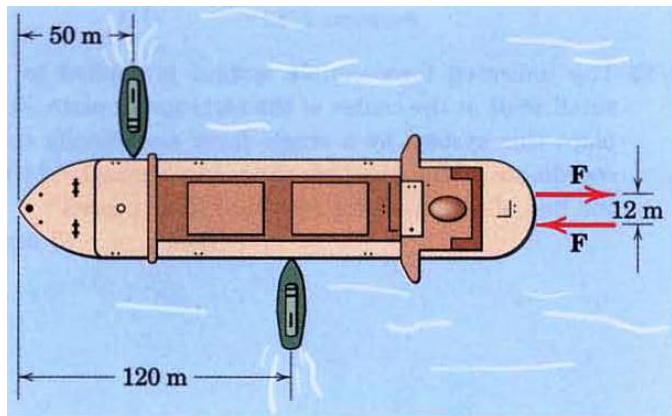
During a steady right turn, a person exerts the forces shown on the steering wheel. Note that each force consists of a tangential component and a radially in ward component. Determine the moment exerted about the steering column at O.

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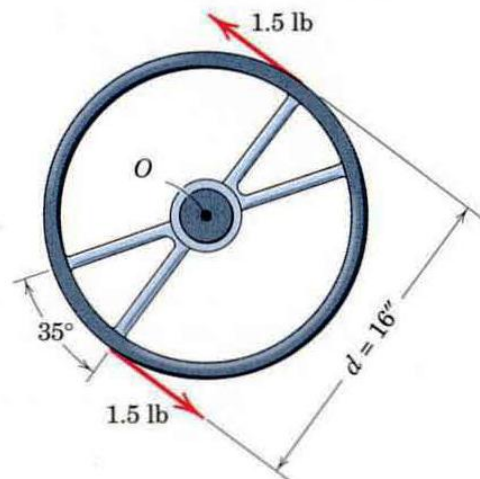
1.8 Resultant of Non Concurrent Coplanar Force

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

$$\mathbf{R} = \Sigma\mathbf{F}$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$

Determine the resultant of the four forces and one couple which act on the plate shown.

Solution. Point O is selected as a convenient reference point for the force–couple system that is to represent the given system.

$[R_x = \Sigma F_x]$ $\qquad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9$ N

$[R_y = \Sigma F_y]$ $\qquad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4$ N

$[R = \sqrt{R_x^2 + R_y^2}]$ $\qquad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3$ N $\qquad$ *Ans.*

$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right]$ $\qquad \theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$ $\qquad$ *Ans.*

$[M_O = \Sigma(Fd)]$ $\qquad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$\qquad\qquad\qquad\qquad\quad = -237$ N·m

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.
We now determine the final line of action of **R** such that **R** alone represents the original system.

$[Rd = |M_O|]$ $\qquad 148.3d = 237 \qquad d = 1.600$ m $\qquad$ *Ans.*

Hence, the resultant **R** may be applied at any point on the line which makes a 63.2° angle with the x-axis and is tangent at point A to a circle of 1.6-m radius with center O, as shown in part b of the figure. We apply the equation $Rd = M_O$ in an absolute-value sense (ignoring any sign of M_O) and let the physics of the situation, as depicted in Fig. a, dictate the final placement of **R**. Had M_O been counterclockwise, the correct line of action of **R** would have been the tangent at point B.

The resultant **R** may also be located by determining its intercept distance b to point C on the x-axis, Fig. c. With R_x and R_y acting through point C, only R_y exerts a moment about O so that

$$R_y b = |M_O| \quad \text{and} \quad b = \frac{237}{132.4} = 1.792 \text{ m}$$

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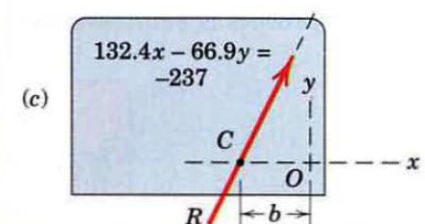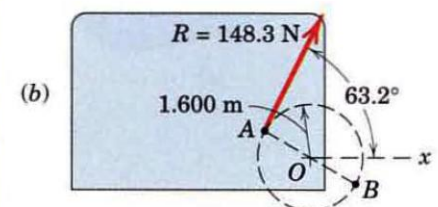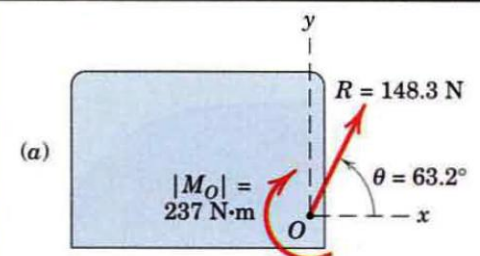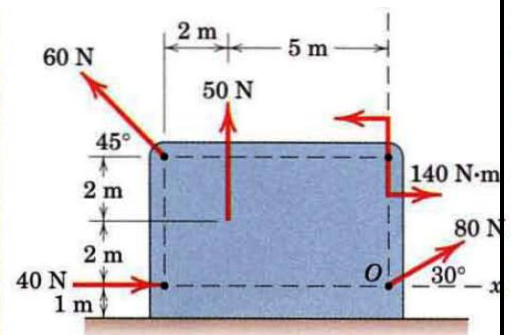
Example

Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.
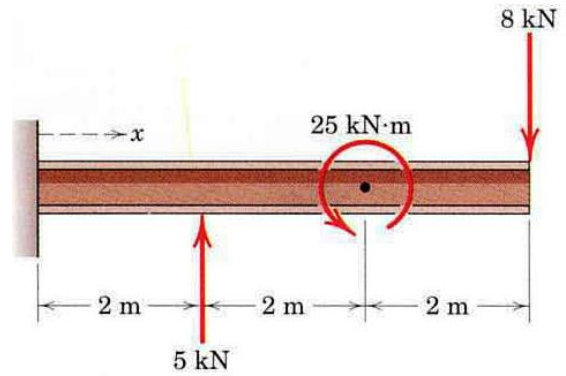
Example

If t he resultant of the two forces and couple M passes through point 0 , determine M if you know the total moment at O equal to zero.

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H.W
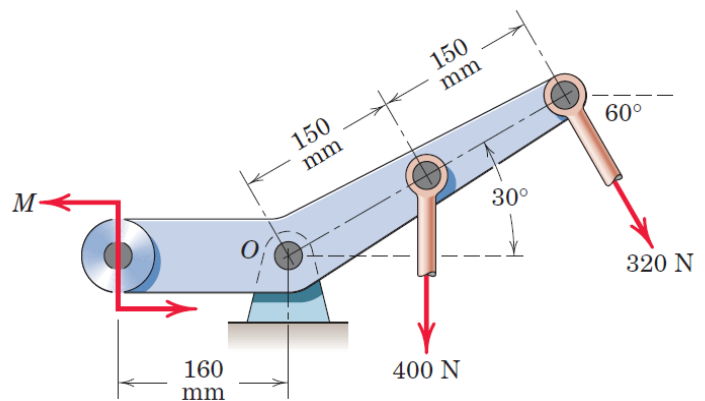
Determine the resultant of the forces and couple applied to the truss and its location respected to **A.**
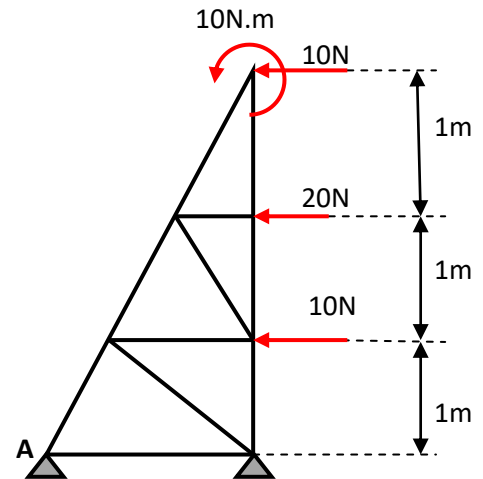
H.W

A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two dimensional problem.

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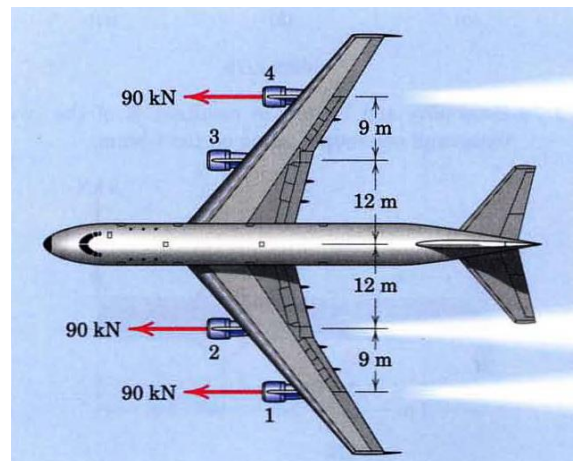
FREE-BODY DIAGRAM EXERCISES

In each of the five following examples, the body to be isolated is shown in the left-hand diagram, and an *incomplete* free-body diagram (FBD) of the isolated body is shown on the right. Add whatever forces are

	Body	Incomplete FBD
1. Bell crank supporting mass *m* with pin support at *A*.		
2. Control lever applying torque to shaft at *O*.		
3. Boom *OA*, of negligible mass compared with mass *m*. Boom hinged at *O* and supported by hoisting cable at *B*.		
4. Uniform crate of mass *m* leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at *A* and fixed pin in smooth slot at *B*.		

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	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ.		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

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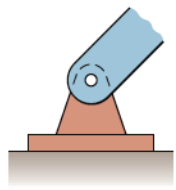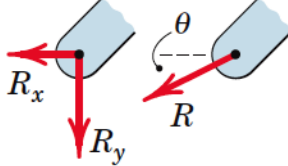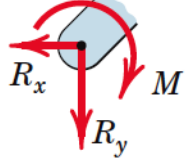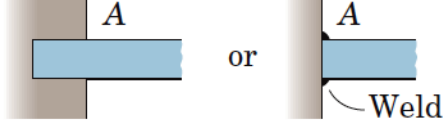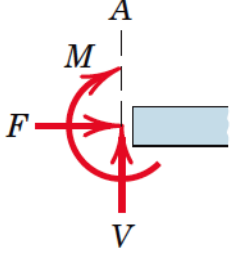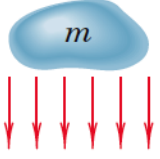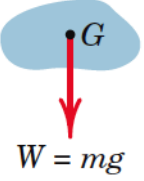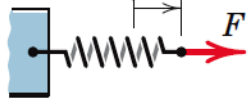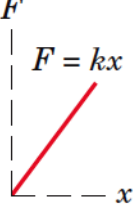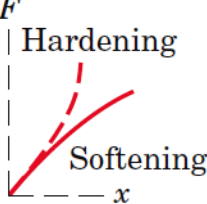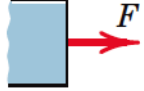
1.9 Equilibrium Conditions

We defined equilibrium as the condition in which the resultant of all forces and moments acting on a body is zero. Stated in another way, a body is in equilibrium if all forces and moments applied to it are in balance

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum M = 0,$$

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible	Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces	Contact force is compressive and is normal to the surface.
3. Rough surfaces	Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support	Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide	Collar or slider free to move along smooth guides; can support force normal to guide only.

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MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)

Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection	Pin free to turn — A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support A or A Weld	A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction m	The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action Neutral position Linear $F = kx$ Nonlinear Hardening Softening	Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

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Example

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.

Solution. The given sketch constitutes the free-body diagram of the isolated
① section of the joint in question and shows the five forces which are in equilibrium.

Solution 1 (scalar algebra). For the x-y axes as shown we have

$[\Sigma F_x = 0]$ $\qquad 8 + T \cos 40° + C \sin 20° - 16 = 0$

$\qquad\qquad\qquad 0.766T + 0.342C = 8$ $\qquad (a)$

$[\Sigma F_y = 0]$ $\qquad T \sin 40° - C \cos 20° - 3 = 0$

$\qquad\qquad\qquad 0.643T - 0.940C = 3$ $\qquad (b)$

Simultaneous solution of Eqs. (a) and (b) produces

$\qquad\qquad T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN}$ $\qquad Ans.$

Solution II (scalar algebra). To avoid a simultaneous solution, we may use axes
② x'-y' with the first summation in the y'-direction to eliminate reference to T. Thus,

$[\Sigma F_{y'} = 0]$ $\quad -C \cos 20° - 3 \cos 40° - 8 \sin 40° + 16 \sin 40° = 0$

$\qquad\qquad C = 3.03 \text{ kN}$ $\qquad Ans.$
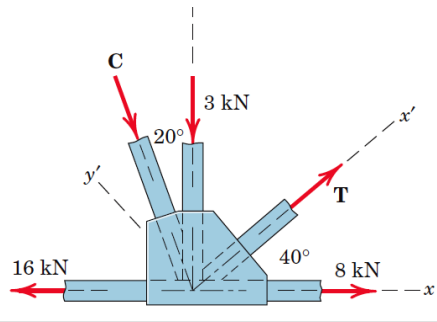
Helpful Hints

① Since this is a problem of concurrent forces, no moment equation is necessary.

② The selection of reference axes to facilitate computation is always an important consideration. Alternatively in this example we could take a set of axes along and normal to the direction of **C** and employ a force summation normal to **C** to eliminate it.

Example

Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5-m I-beam with a mass of 95 kg per meter of length.

Algebraic solution. The system is symmetrical about the vertical x-y plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components. The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN and acts through its center. Note that there are three unknowns A_x, A_y, and T, which may be found from the three equations of equilibrium. We begin with a moment equation about A, which eliminates two of the three unknowns from the equation. In applying the moment equation about A, it is simpler to consider the moments of the x- and y-components of **T** than it is to compute the perpendicular distance from **T** to A. Hence, with the counterclockwise sense as positive we write

$[\Sigma M_A = 0]$ $\qquad (T \cos 25°)0.25 + (T \sin 25°)(5 - 0.12)$
$\qquad\qquad\qquad - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$

from which $\qquad\qquad T = 19.61 \text{ kN}$ $\qquad Ans.$

Equating the sums of forces in the x- and y-directions to zero gives

$[\Sigma F_x = 0]$ $\qquad A_x - 19.61 \cos 25° = 0 \qquad\qquad A_x = 17.77 \text{ kN}$

$[\Sigma F_y = 0]$ $\quad A_y + 19.61 \sin 25° - 4.66 - 10 = 0 \qquad A_y = 6.37 \text{ kN}$

$[A = \sqrt{A_x{}^2 + A_y{}^2}] \quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN}$ $\qquad Ans.$

Free-body diagram
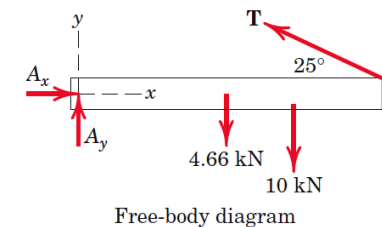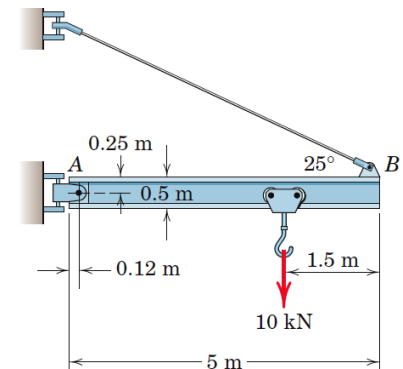
Helpful Hints

① The justification for this step is Varignon's theorem, explained in Art. 2/4. Be prepared to take full advantage of this principle frequently.

② The calculation of moments in two-dimensional problems is generally handled more simply by scalar alge-

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Example

The mass center G of the 1400-kg rear-engine car is located as shown in the figure. Determine the normal force under each tire when the car is in equilibrium. State any assumptions.

←— 1386 mm —→| 964 mm |←
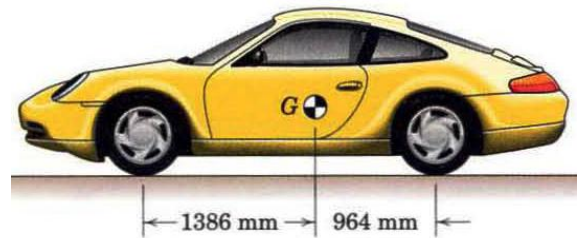
Example

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.

←— 5.6 m —→|←— 2.4 m —→|
A   B
220 kg

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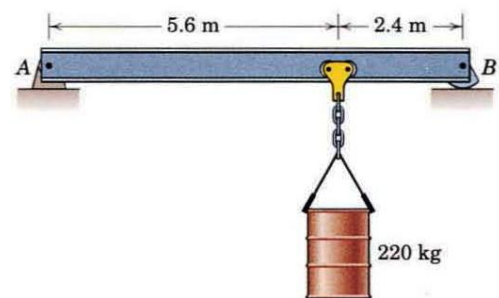
Example

The indicated location of the center of gravity of the 3600-1b pickup truck is for the un laden condition. If a load whose center of gravity is $x = 16$ in. behind the rear axle is added to the truck, determine the load weight WL for which the normal forces under the front and rear wheels are equal
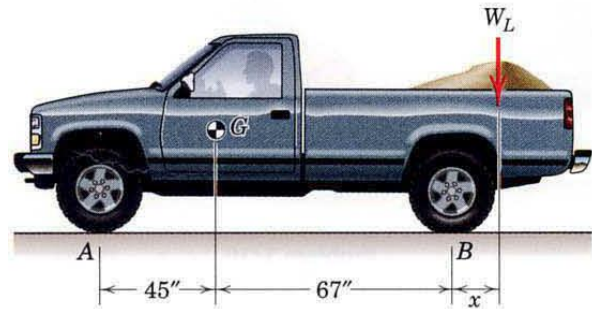
Example

A man is holding an 8-lb weight in her hand with the entire arm held horizontally as shown in the figure. A tensile force in the deltoid muscle prevents the arm from rotating about the shoulder joint O ; this force acts at the 21° angle shown. Determine the force exerted by the deltoid muscle on the upper arm at A and the .x- and y-components of the force reaction at the shoulder joint O. The weight of the upper arm is $V_U = 4.1$ lb, the weight of t he lower arm is W_L, = 2.4 lb, and the weight of the hand is $W_H = 0.9$ lb; all these weights act at the locations shown in the figure.   ***Ans.*** $F_D = 160.2$ lb, $O_x = 149.5$ Ib, $O_y = - 42.2$ lb

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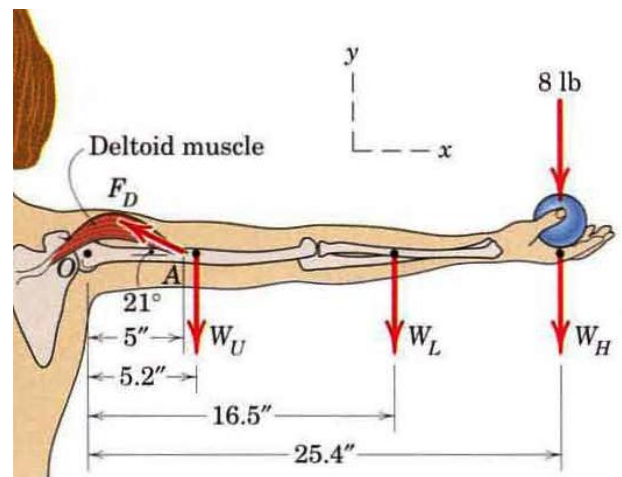
Example

The man pushes the lawn mower at a steady speed with a force P that is parallel to the incline. The mass of the mower with attached grass bag is 50 kg with mass center at G. If $\theta = 15°$, determine the normal forces N_B and N_c under each pair of wheels B and C. Neglect friction. Compare with the normal forces for the conditions of $\theta = 0$ and $P = O$.

Ans. $N_B = 214$ N, $N_C = 260$ N With $\theta = P = 0$: $N_B = 350$ N, $N_c = 140.1$ N
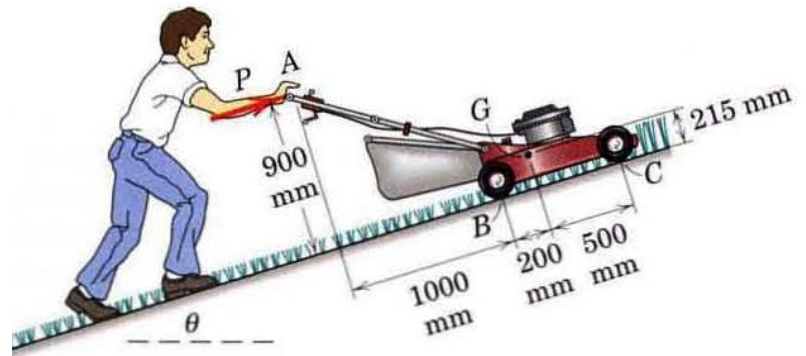
Example

The uniform bar with end rollers weighs 60 lb and is supported by the horizontal and vertical surfaces and by the wire *AC*. Calculate the tension *T* in the wire and the reactions against the rollers at *A* and at *B*.

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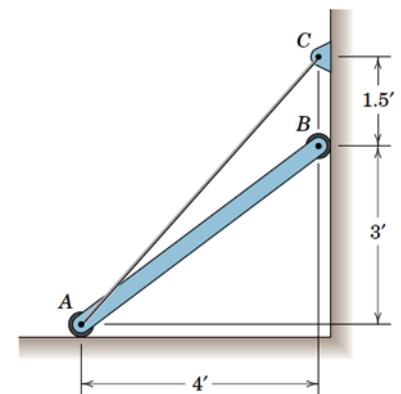
Example

The device shown in the figure is useful for lifting drywall panels into position prior to fastening to the stud wall. Estimate the magnitude P of the force required to lift the 25-kg panel. State any assumptions.
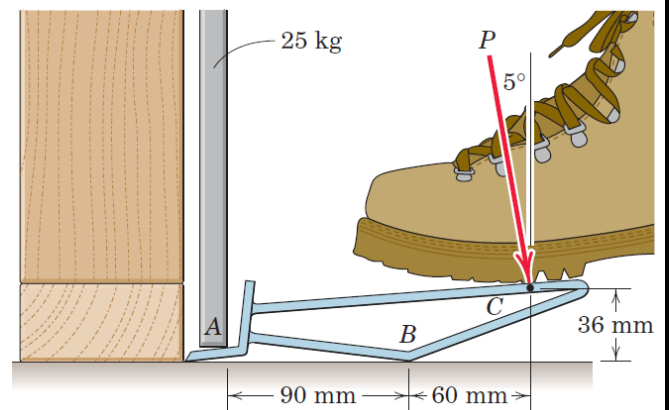
1. 10   Analysis of Truss

Figure 4/1

Pratt

Howe

Warren

K

Baltimore

Commonly Used Bridge Trusses

Fink

Pratt

Howe

Warren

Commonly Used Roof Trusses

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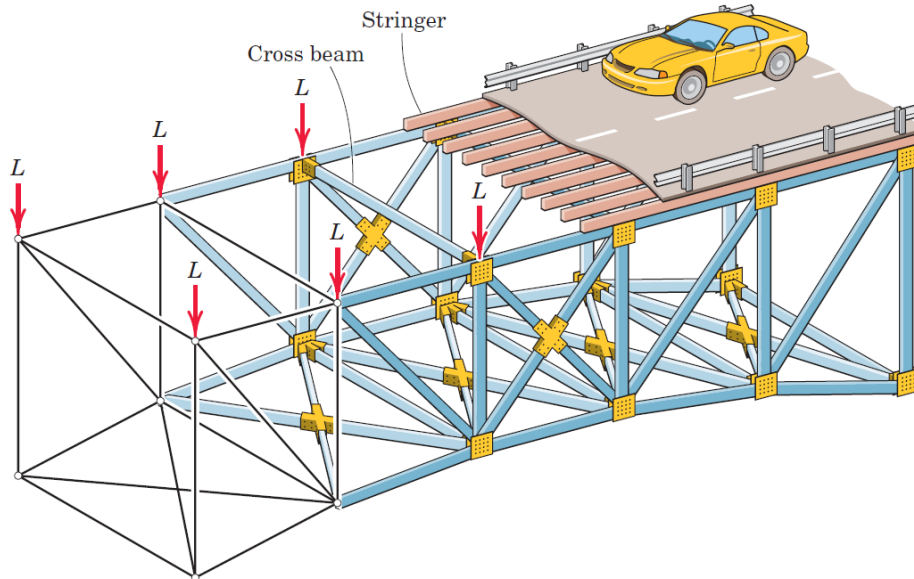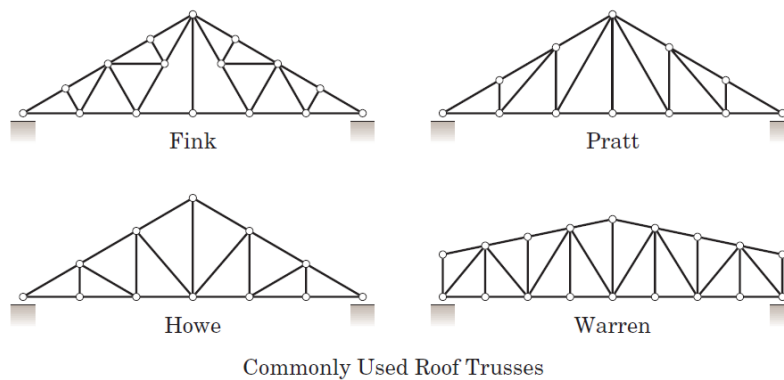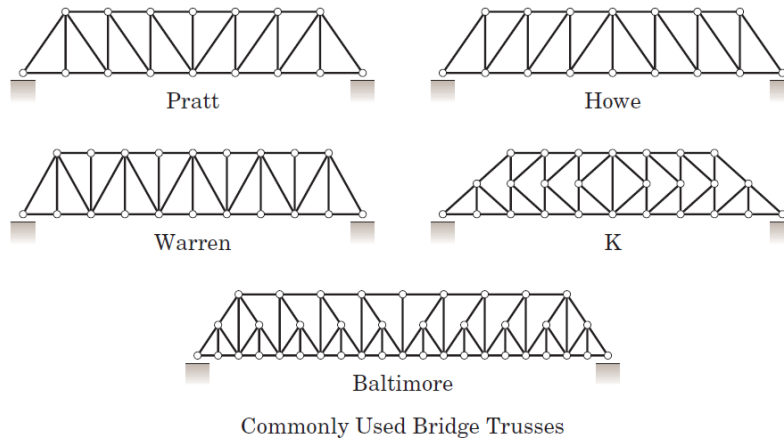
There are two methods to analysis the forces system in truss structure
  1- Method of joint
  2- Method of section

1- Method of Joints

$$\sum F_x = 0, \qquad\qquad \sum F_y = 0$$

Example

Determine the forces in member CG, FG and CH in the truss of

$$\sum F_x = 0$$

$$A_x = 0$$

$$\sum F_y = 0$$

$$A_y - 2000 - 4000 - 2000 + E_y = 0$$

By symmetric the truss and the distribution of loads

$$A_y = E_y = \frac{8000}{2} = 4000 Lb \uparrow$$

At joint A

$$\uparrow \sum F_y = 0 = 4000 - \frac{3}{5} F_{AB} = 0 \rightarrow \qquad F_{AB} = \frac{2000}{3} Lb$$

$$\sum F_x = 0 = F_{AB} \cos\theta - F_{AH} = 0 \rightarrow \qquad F_{AH} = \frac{2000}{3} \times \frac{4}{5} = \frac{16000}{3} Lb$$

At joint B

$$\sum F_x = 0 \rightarrow \qquad = \frac{2000}{3} \times \frac{4}{5} - F_{BC} \rightarrow \quad F_{BC} = 4000 \frac{16000}{3} Lb$$

$$\uparrow \sum F_y = 0 = \qquad \frac{2000}{3} \times \frac{3}{5} + F_{BH} - 2000 = 0 \rightarrow$$

$$F_{BH} = 2000 Lb$$

At joint H

$$\uparrow \sum F_y = 0 = \qquad \frac{3}{5} \times F_{HC} - 2000 = 3333.3$$

$$\sum F_x = 0 \rightarrow \qquad = \frac{-16000}{3} - \frac{10000}{3}\frac{4}{5} + F_{HG} \rightarrow \quad F_{HG} = 8000 Lb$$

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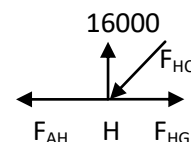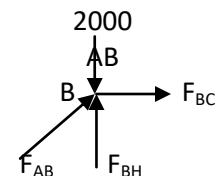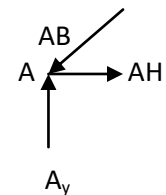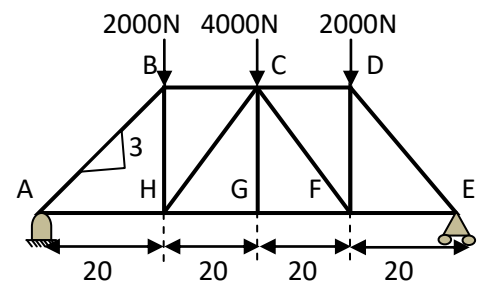
Compute the force in each member of the loaded cantilever truss by the method of joints.

Solution. If it were not desired to calculate the external reactions at D and E, the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give

$[\Sigma M_E = 0]$ $\qquad$ $5T - 20(5) - 30(10) = 0$ $\qquad$ $T = 80$ kN

$[\Sigma F_x = 0]$ $\qquad$ $80 \cos 30° - E_x = 0$ $\qquad$ $E_x = 69.3$ kN

$[\Sigma F_y = 0]$ $\qquad$ $80 \sin 30° + E_y - 20 - 30 = 0$ $\qquad$ $E_y = 10$ kN

Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence. There should be no question about the correct direction of the forces on joint A. Equilibrium requires

$[\Sigma F_y = 0]$ $\quad$ $0.866AB - 30 = 0$ $\quad$ $AB = 34.6$ kN T $\qquad$ *Ans.*

$[\Sigma F_x = 0]$ $\quad$ $AC - 0.5(34.6) = 0$ $\quad$ $AC = 17.32$ kN C $\qquad$ *Ans.*

where T stands for tension and C stands for compression.

Joint B must be analyzed next, since there are more than two unknown forces on joint C. The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from

$[\Sigma F_y = 0]$ $\quad$ $0.866BC - 0.866(34.6) = 0$ $\quad$ $BC = 34.6$ kN C $\qquad$ *Ans.*

$[\Sigma F_x = 0]$ $\quad$ $BD - 2(0.5)(34.6) = 0$ $\quad$ $BD = 34.6$ kN T $\qquad$ *Ans.*

Joint C now contains only two unknowns, and these are found in the same way as before:

$[\Sigma F_y = 0]$ $\quad$ $0.866CD - 0.866(34.6) - 20 = 0$

$\qquad\qquad$ $CD = 57.7$ kN T $\qquad$ *Ans.*

$[\Sigma F_x = 0]$ $\quad$ $CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$\qquad\qquad$ $CE = 63.5$ kN C $\qquad$ *Ans.*

Finally, from joint E there results

$[\Sigma F_y = 0]$ $\quad$ $0.866DE = 10$ $\quad$ $DE = 11.55$ kN C $\qquad$ *Ans.*

and the equation $\Sigma F_x = 0$ checks.

Note that the weights of the truss members have been neglected in comparison with the external loads.
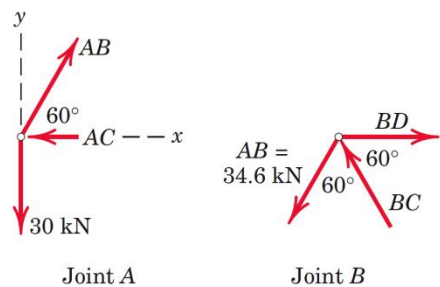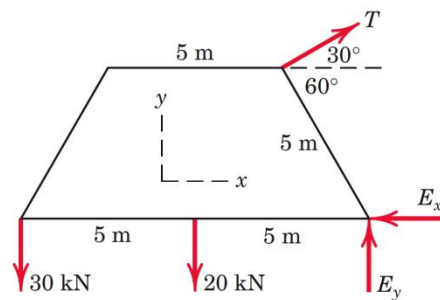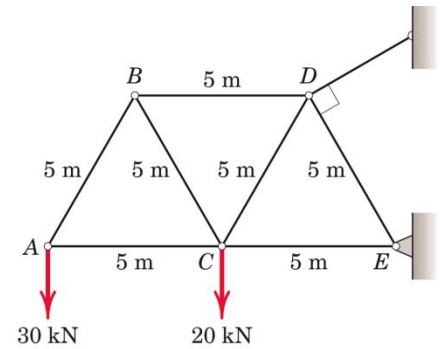
Helpful Hint

① It should be stressed that the tension/compression designation refers to the *member*, not the joint. Note that we draw the force arrow on the same side of the joint as the member which exerts the force. In this way tension (arrow away from the joint) is distinguished from compression (arrow toward the joint).

Joint C $\qquad$ Joint E

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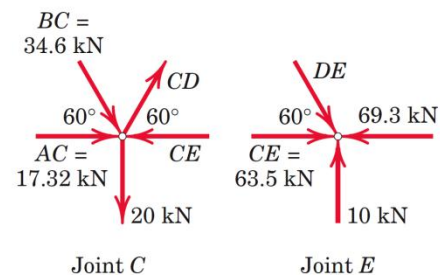
Example

Determine the force in members HI, FG and BC of the truss.

Ans: $F_{HI} = 12000Lb$, $F_{FG} = (12000/8)$ Lb

and $F_{BC} = 9000/6Lb$

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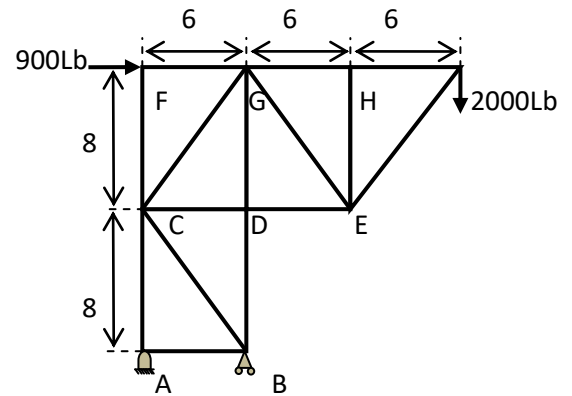
2-   Section Method

~~SAMPLE PROBLEM 4/6~~

Calculate the forces induced in members KL, CL, and CB by the 20-ton load on the cantilever truss.

Solution.   Although the vertical components of the reactions at A and M are statically indeterminate with the two fixed supports, all members other than AM are statically determinate. We may pass a section directly through members KL, CL, and CB and analyze the portion of the truss to the left of this section as a statically determinate rigid body.

The free-body diagram of the portion of the truss to the left of the section is shown. A moment sum about L quickly verifies the assignment of CB as compression, and a moment sum about C quickly discloses that KL is in tension. The direction of CL is not quite so obvious until we observe that KL and CB intersect at a point P to the right of G. A moment sum about P eliminates reference to KL and CB and shows that CL must be compressive to balance the moment of the 20-ton force about P. With these considerations in mind the solution becomes straightforward, as we now see how to solve for each of the three unknowns independently of the other two.

Summing moments about L requires finding the moment arm $\overline{BL} = 16 + (26 - 16)/2 = 21$ ft. Thus,

$[\Sigma M_L = 0]$ $20(5)(12) - CB(21) = 0$ $CB = 57.1$ tons C *Ans.*

Next we take moments about C, which requires a calculation of $\cos\theta$. From the given dimensions we see $\theta = \tan^{-1}(5/12)$ so that $\cos\theta = 12/13$. Therefore,

$[\Sigma M_C = 0]$ $20(4)(12) - \frac{12}{13}KL(16) = 0$ $KL = 65$ tons T *Ans.*

Finally, we may find CL by a moment sum about P, whose distance from C is given by $\overline{PC}/16 = 24/(26 - 16)$ or $\overline{PC} = 38.4$ ft. We also need β, which is given by $\beta = \tan^{-1}(\overline{CB}/\overline{BL}) = \tan^{-1}(12/21) = 29.7°$ and $\cos\beta = 0.868$. We now have

$[\Sigma M_p = 0]$ $20(48 - 38.4) - CL(0.868)(38.4) = 0$

$CL = 5.76$ tons C *Ans.*
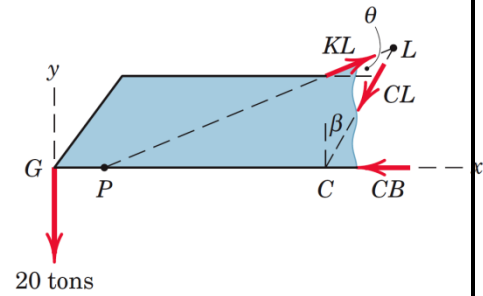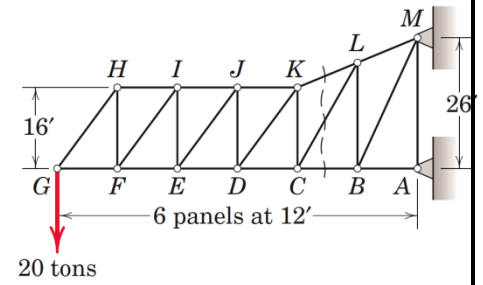
Helpful Hints

① We note that analysis by the method of joints would necessitate working with eight joints in order to calculate the three forces in question. Thus, the method of sections offers a considerable advantage in this case.

② We could have started with moments about C or P just as well.

③ We could also have determined CL by a force summation in either the x- or y-direction.

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SAMPLE PROBLEM 4/4

Calculate the force in member DJ of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.

Solution. It is not possible to pass a section through DJ without cutting four members whose forces are unknown. Although three of these cut by section 2 are concurrent at J and therefore the moment equation about J could be used to obtain DE, the force in DJ cannot be obtained from the remaining two equilibrium principles. It is necessary to consider first the adjacent section 1 before analyzing section 2.

The free-body diagram for section 1 is drawn and includes the reaction of 18.33 kN at A, which is previously calculated from the equilibrium of the truss as a whole. In assigning the proper directions for the forces acting on the three cut members, we see that a balance of moments about A eliminates the effects of CD and JK and clearly requires that CJ be up and to the left. A balance of moments about C eliminates the effect of the three forces concurrent at C and indicates that JK must be to the right to supply sufficient counterclockwise moment. Again it should be fairly obvious that the lower chord is under tension because of the bending tendency of the truss. Although it should also be apparent that the top chord is under compression, for purposes of illustration the force in CD will be arbitrarily assigned as tension.

By the analysis of section 1, CJ is obtained from

$[\Sigma M_A = 0]$ $\qquad$ $0.707CJ(12) - 10(4) - 10(8) = 0$ $\qquad$ $CJ = 14.14$ kN C

In this equation the moment of CJ is calculated by considering its horizontal and vertical components acting at point J. Equilibrium of moments about J requires

$[\Sigma M_J = 0]$ $\qquad$ $0.894CD(6) + 18.33(12) - 10(4) - 10(8) = 0$

$\qquad\qquad CD = -18.63$ kN

The moment of CD about J is calculated here by considering its two components as acting through D. The minus sign indicates that CD was assigned in the wrong direction.

Hence, $\qquad\qquad\qquad CD = 18.63$ kN C

From the free-body diagram of section 2, which now includes the known value of CJ, a balance of moments about G is seen to eliminate DE and JK. Thus,

$[\Sigma M_G = 0]$ $\qquad$ $12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$

$\qquad\qquad DJ = 16.67$ kN T $\qquad\qquad\qquad\qquad\qquad$ *Ans.*

Again the moment of CJ is determined from its components considered to be acting at J. The answer for DJ is positive, so that the assumed tensile direction is correct.

An alternative approach to the entire problem is to utilize section 1 to determine CD and then use the method of joints applied at D to determine DJ.
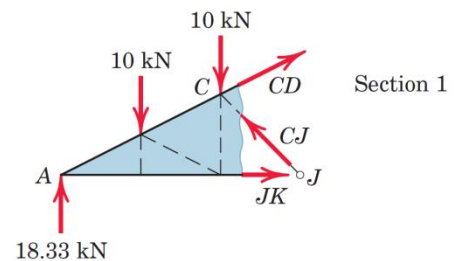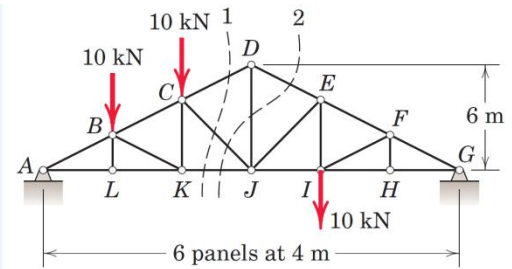
Section 1

Helpful Hints

① There is no harm in assigning one or more of the forces in the wrong direction, as long as the calculations are consistent with the assumption. A negative answer will show the need for reversing the direction of the force.

② If desired, the direction of CD may be changed on the free-body diagram and the algebraic sign of CD reversed in the calculations, or else the work may be left as it stands with a note stating the proper direction.
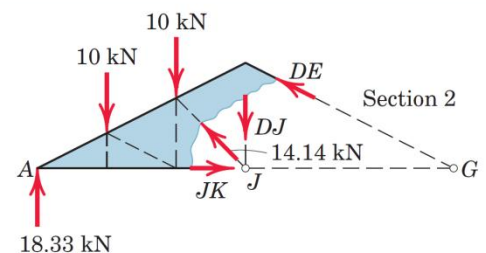
Section 2

③ Observe that a section through members CD, DJ, and DE could be taken which would cut only three unknown members. However, since the forces in these three members are all concurrent at D, a moment equation about D would yield no information

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Example
Find the internal forces in each member

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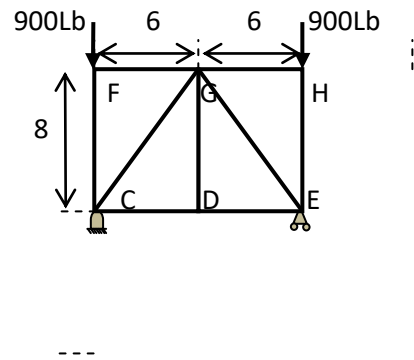
4/48 Compute the force in member *GM* of the loaded truss.
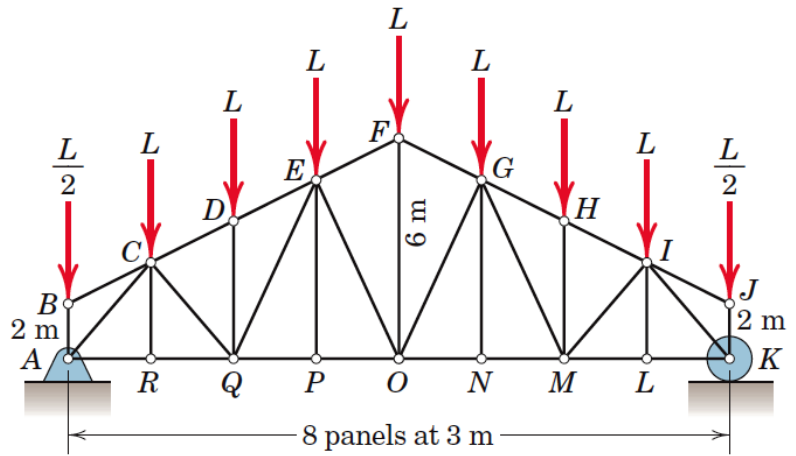
Problem 4/48

4/49 Determine the force in member *BE* of the loaded truss.
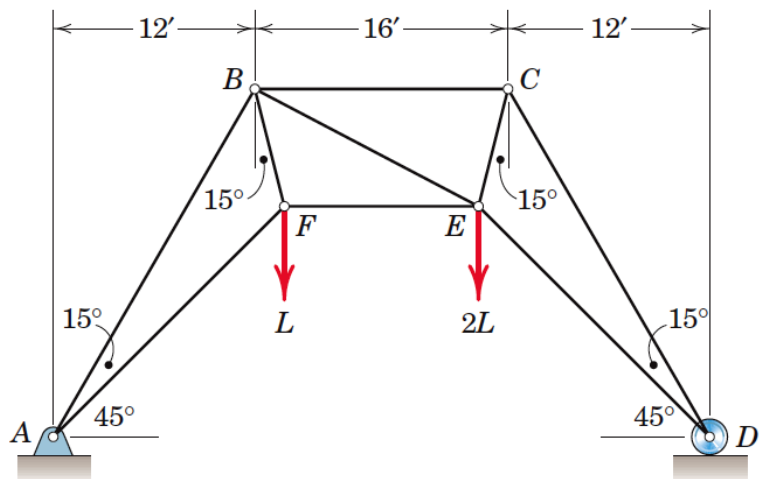
Problem 4/49

4/50 Determine the forces in members *DJ* and *EJ* of the loaded truss.

6 panels at 8 m

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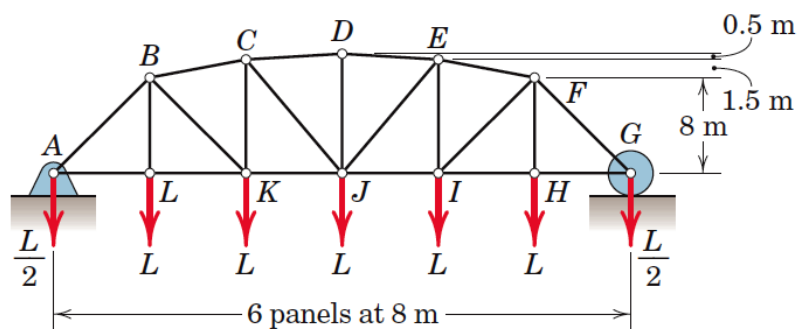
1.11 Friction

Dry friction resists relative lateral motion of two solid surfaces in contact. Dry friction is subdivided into *static friction* ("stiction") between non-moving surfaces, and *kinetic friction* between moving surfaces.

A- Friction in a Horizontal Surface

Friction force is parallel and opposite to the movement
Where ; P : is pulling force
        N : is normal reaction
        F : friction force

(a)

(b)

(c)

(d)

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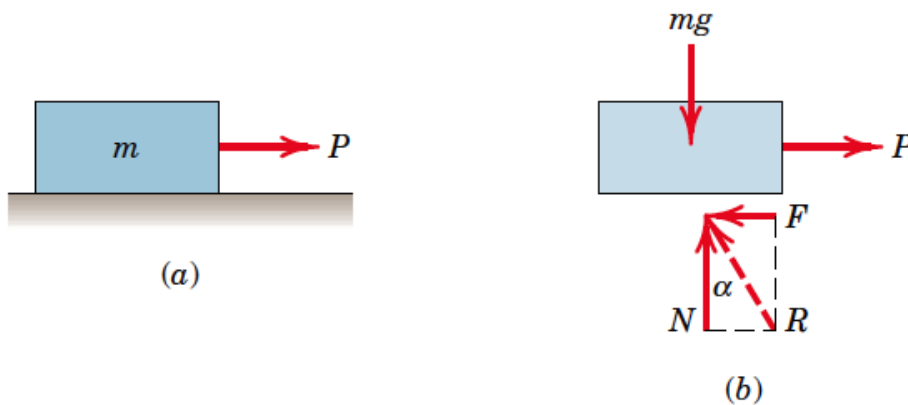
1- In case of Static

$$\sum \overset{\rightarrow +}{F_x} = 0 \rightarrow \quad P - F = 0 \rightarrow \quad \therefore P = F$$

$$\uparrow \sum F_y = 0 \rightarrow \quad N - mg = 0$$

At maximum friction

For a given pair of mating surfaces the experiment shows that this **maximum value of static friction F$_{max}$** is proportional to the normal force *N*. Thus , we may write
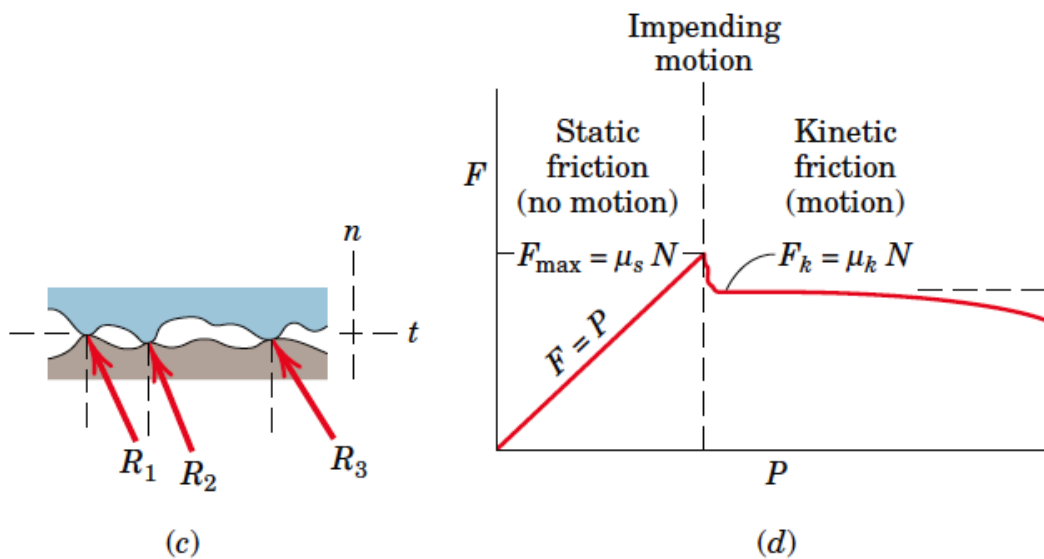
$$F_{max} = N \times \mu_s$$

Where **μ$_s$** is a *coefficient of static friction.*

2- In Case dynamic

After slippage occurs

$$F_K = N \times \mu_k$$

where **μ$_k$** is the *coefficient of kinetic friction*

B- In Inclined Surface

$$\tan \alpha = \frac{F}{N}$$

at maximum value of α (before the body stars in movement)

$$\tan \alpha = \mu_s$$
In moving

$$\tan \alpha = \mu_k$$

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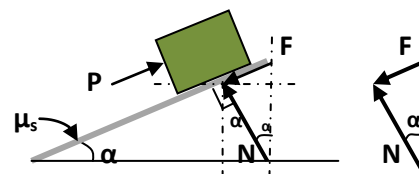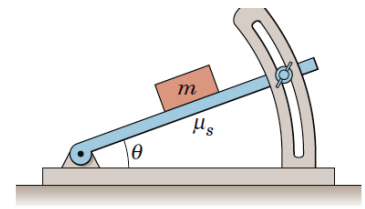Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.

Solution. The free-body diagram of the block shows its weight $W = mg$, the normal force N, and the friction force F exerted by the incline on the block. The friction force acts in the direction to oppose the slipping which would occur if no friction were present.
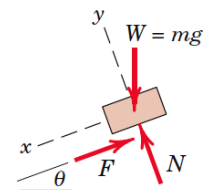
❶ Equilibrium in the x- and y-directions requires

$[\Sigma F_x = 0]$ $\quad\quad mg \sin \theta - F = 0 \quad\quad F = mg \sin \theta$

$[\Sigma F_y = 0]$ $\quad\quad -mg \cos \theta + N = 0 \quad\quad N = mg \cos \theta$

Dividing the first equation by the second gives $F/N = \tan \theta$. Since the maximum angle occurs when $F = F_{max} = \mu_s N$, for impending motion we have

❷ $\quad\quad\quad \mu_s = \tan \theta_{max} \quad\quad \text{or} \quad\quad \theta_{max} = \tan^{-1} \mu_s \quad\quad\quad Ans.$

Helpful Hints

❶ We choose reference axes along and normal to the direction of F to avoid resolving both F and N into components.

❷ This problem describes a very simple way to determine a static coefficient of friction. The maximum value of θ is known as the *angle of repose*.

SAMPLE PROBLEM 6/2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.

Solution. The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give

$[\Sigma F_y = 0]$ $\quad\quad N - 981 \cos 20° = 0 \quad\quad N = 922$ N

$[F_{max} = \mu_s N]$ $\quad\quad F_{max} = 0.30(922) = 277$ N

$[\Sigma F_x = 0]$ $\quad m_0(9.81) - 277 - 981 \sin 20° = 0 \quad\quad m_0 = 62.4$ kg $\quad\quad Ans.$

❶ The minimum value of m_0 is determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the x-direction requires

$[\Sigma F_x = 0]$ $\quad m_0(9.81) + 277 - 981 \sin 20° = 0 \quad\quad m_0 = 6.01$ kg $\quad\quad Ans.$

Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

In both cases equilibrium requires that the resultant of F_{max} and N be concurrent with the 981-N weight and the tension T.
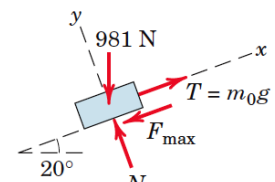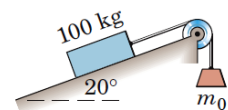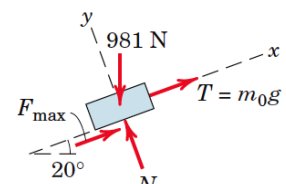
Helpful Hint

❶ We see from the results of Sample Problem 6/1 that the block would slide down the incline without the restraint of attachment to m_0 since tan 20° > 0.30. Thus, a value of m_0 will be required to maintain equilibrium.

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SAMPLE PROBLEM 6/3

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, $P = 500$ N and, second, $P = 100$ N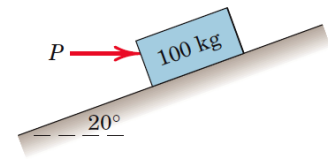. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.

Solution. There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P. It is therefore necessary that we make an assumption, so we will take the friction force to be up the plane, as shown by the solid arrow. From the free-body diagram a balance of forces in both x- and y-directions gives

$[\Sigma F_x = 0]$ $\qquad\qquad P \cos 20° + F - 981 \sin 20° = 0$

$[\Sigma F_y = 0]$ $\qquad\qquad N - P \sin 20° - 981 \cos 20° = 0$
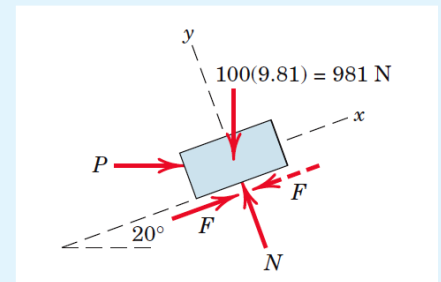
Case I. $P = 500$ N

Substitution into the first of the two equations gives

$$F = -134.3 \text{ N}$$

The negative sign tells us that *if* the block is in equilibrium, the friction force acting on it is in the direction opposite to that assumed and therefore is down the plane, as represented by the dashed arrow. We cannot reach a conclusion on the magnitude of F, however, until we verify that the surfaces are capable of supporting 134.3 N of friction force. This may be done by substituting $P = 500$ N into the second equation, which gives

$$N = 1093 \text{ N}$$

The maximum static friction force which the surfaces can support is then

$[F_{max} = \mu_s N]$ $\qquad\qquad F_{max} = 0.20(1093) = 219 \text{ N}$

Since this force is greater than that required for equilibrium, we conclude that the assumption of equilibrium was correct. The answer is, then,

$$F = 134.3 \text{ N down the plane} \qquad\qquad Ans.$$

Case II. $P = 100$ N

Substitution into the two equilibrium equations gives

$$F = 242 \text{ N} \qquad N = 956 \text{ N}$$

But the maximum possible static friction force is

$[F_{max} = \mu_s N]$ $\qquad\qquad F_{max} = 0.20(956) = 191.2 \text{ N}$

It follows that 242 N of friction cannot be supported. Therefore, equilibrium cannot
1 exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the motion down the plane. Hence, the answer is

$[F_k = \mu_k N]$ $\qquad\qquad F = 0.17(956) = 162.5 \text{ N up the plane} \qquad\qquad Ans.$

Helpful Hint

1 We should note that even though ΣF_x is no longer equal to zero, equilibrium does exist in the y-direction, so that $\Sigma F_y = 0$. Therefore, the normal force N is 956 N whether or not the block is in equilibrium.

*Assistant Prof. Dr. Arshed Abdulhamed*

Example

The 700-N force is applied to the 100-kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the block.

$P = 700$ N

100 kg

$30°$

$\mu_s = 0.80$
$\mu_k = 0.60$
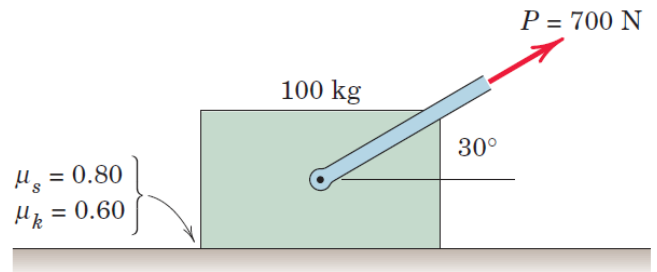
Example

The designer of a ski resort wishes to have a portion of a beginner's slope on which the skier's speed will remain fairly constant. Tests indicate the average coefficients of friction between skis and snow to be $\mu_s = 0.10$ and $\mu_k = 0.08$. What should be the slope angle θ of the constant speed section?

θ

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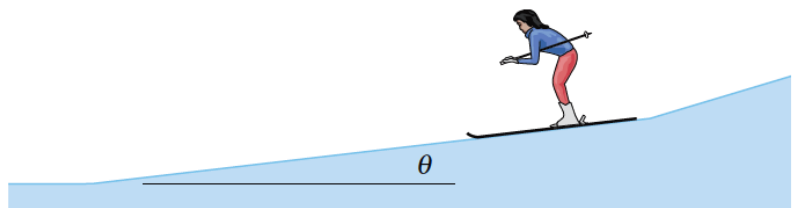
Example

The coefficients of static and kinetic friction between the 100kg block and the inclined plane are 0.30 and 0.20, respectively. Determine (0) the friction force F acting on the block when P is applied with a magnitude of 200 N to the block at rest, *(b)* the force P required to initiate motion up the incline from rest. and *(c)* the friction force F acting on the block if P 600 N.
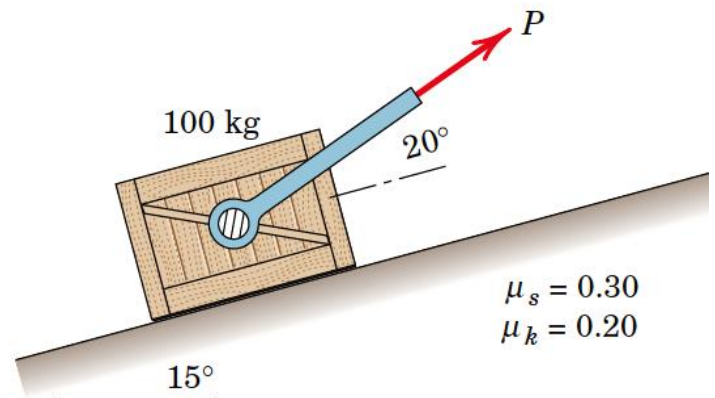
Example
Determine the distances to which the 90-kg painter can climb without causing the 4-m ladder to slip at its lower end A. The top of the I5-kg ladder has a small roller, and at the ground the coefficient of static friction is 0.25. The mass center of the painter is directly above her feet.

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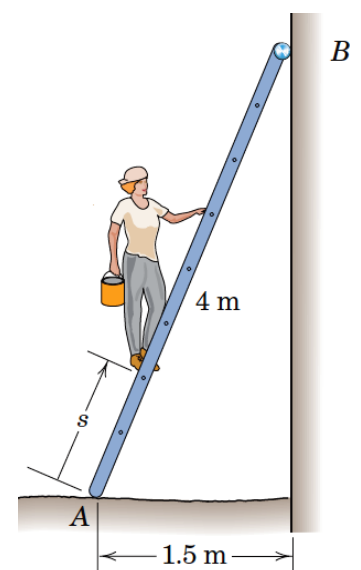
Example

A loaded wheelbarrow is placed on a rough incline. The combined weight of the wheelbarrow and its load acts at the center of gravity G. (*a*) Determine the maximum angle θ for which the wheelbarrow will not slip. (*b*) Repeat if the wheelbarrow faces uphill. Neglect all friction associated with the front wheel B.

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